ICM: Sunyaev-Zel’dovich Effects

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Introduction

- SZ-Effect connects two important cosmological entities

- The effect was first described by Sunyaev & Zel’dovich (1970,1972)
- Former (hypothetical) work was done by Weymann (1966) and Sunyaev & Zel’dovich (1969). Impact of hot intergalactic gas on the CMB.
- Today the SZ-Effect has become a powerful cosmological tool
SZ-Mechanism: Inverse Compton-Scattering
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Microwave Background Photon

Hot Cluster Gas

Energetic Electron

Blue Shifted Microwave Photon

Observer

Sunyaev-Zeldovich Effect

Electron

Photon

Photon

Photon

"Cluster of Galaxies" Sumo Arena
Definitions

\[ \mu = \cos \Theta, \quad \mu' = \cos \Theta', \quad \beta = \frac{v_e}{c}, \quad x = \frac{h\nu}{kT_{\text{CMB}}}, \quad x_e = \frac{h\nu}{kT_e} \]

\[ \tau_e \equiv \int n_e \sigma_T \, dl, \quad y = \tau_e \Theta, \quad \Theta = \frac{kT_e}{m_e c^2} \]
Inverse - Compton Scattering

- The Thomson-limit \((E_{e\rightarrow} \approx E'_{e\rightarrow})\) yields:
  \[
p(\mu) d\mu = \left(2\gamma^4 (1 - \beta \mu)^3\right)^{-1} d\mu
  \]
  \[
  \phi(\mu', \mu) d\mu' = \frac{3}{8} \left(1 + \mu^2 \mu'^2 + \frac{1}{2} (1 - \mu^2) (1 - \mu'^2)\right) d\mu'.
  \]

- The frequency of the scattered photon is then:
  \[
  \nu'' = \nu \left(1 + \beta \mu'\right) (1 - \beta \mu)^{-1}.
  \]

- We define for describing the frequency shift:
  \[
  s = \log \left(\nu'' / \nu\right)
  \]

- Probability for a scattering process with shift \(s\) by electron with velocity \(\beta c\):
  \[
P(s, \beta) = \frac{3}{16\gamma^4\beta} \int_{\mu_1}^{\mu_2} \left(1 + \beta \mu'\right) \left(1 + \mu^2 \mu'^2 + \frac{1}{2} (1 - \mu^2) (1 - \mu'^2)\right) (1 - \beta \mu)^{-3} d\mu
  \]
The Intensity Spectrum

- Electrons are distributed in velocity-space:
  \[ P_1(s) = \int_{\beta_{\text{lim}}}^{1} p_e(\beta) d\beta P(s, \beta) \]

- With the Planckian spectrum of the CMB \( I_0(\nu) \) and the spectrum after scattering:
  \[ I(\nu) = \int_{-\infty}^{\infty} P_1(s) I_0(\nu_0) ds, \]
  \[ \Rightarrow \Delta I(\nu) = \frac{2\hbar c^2}{e^{h\nu_0/kT_{\text{CMB}}} - 1} \int P_1(s) ds \left( \frac{v_0^3}{e^{h\nu/kT_{\text{CMB}}} - 1} - \frac{v^3}{e^{h\nu/kT_{\text{CMB}}} - 1} \right). \]

- Taking into account multiple scattering processes (to 1st order):
  \[ \Delta I(\nu) = \frac{2\hbar}{c^2} T_e \int P_1(s) ds \left( \frac{v_0^3}{e^{h\nu_0/kT_{\text{CMB}}} - 1} - \frac{v^3}{e^{h\nu/kT_{\text{CMB}}} - 1} \right). \]

- Translated to occupation number representation:
  \[ \Delta n(x) = T_e \int_{-\infty}^{\infty} \left[ n(xe^s) - n(x) \right] P_1(s) ds \]
Non-Relativistic Limit

- In the non-relativistic limit the occupation-number distribution is given by the Kompaneets equation

\[
\frac{\partial n}{\partial y} = \frac{1}{x_e^2} \frac{\partial}{\partial x_e} x^4 e \left( \frac{\partial n}{\partial x_e} + n + n^2 \right)
\]

\[
\frac{\partial n}{\partial y} = \frac{1}{x_e^2} \frac{\partial}{\partial x_e} x^4 e \frac{\partial n}{\partial x_e} \quad \text{for small } x_e
\]

- Translating this again to intensity representation yields the non-relativistic scattering kernel

\[
P_k = \frac{1}{\sqrt{4\pi} y} \exp \left( -\frac{(s+3y)^2}{4y} \right)
\]

- One finds the simple analytical expression for the thermal SZ-effect:

\[
\Rightarrow \Delta I(\nu) = \frac{2(kT_{\text{CMB}})^3}{(hc)^2} yg(x)
\]

with the spectral function

\[
g(x) = \frac{x^4 e^x}{(e^x-1)^2} \left[ \frac{x(e^x+1)}{e^x-1} - 4 \right] \quad \text{(crossover frequency of } \sim 217 \, \text{GHz}).
\]
The kinematic SZ-Effect also takes into account the cluster velocity wrt the CMB rest frame.

In the non-relativistic limit the kinematic SZE is a simple temperature-amplitude distortion:

\[ \frac{\Delta T_{CMB}}{T_{CMB}} = -\tau e/\beta_{pec}. \]

Relativistic corrections take into account the Lorentz boost of the electrons and contain higher order terms in \( \beta_{pec} \).

For detecting the kinematic effect the crossover frequency is
Note:
Left panel: Effect of a cluster with 1000 times the usual mass.
Relativistic Corrections

- Relativistic corrections become important at the Wien side of the spectrum.
  Of special interest are changes to the crossover frequency (near the transition to Wien spectrum).
- Also important are the rel. corrections in clusters with high-temperature ICM.
- kSZE becomes frequency dependent
- Different approaches are possible:
  1. Relativistic Maxwellian distribution of the electrons (Rephaeli 1995).
  3. LT of the photon transfer equation (Sazonov & Sunyaev 1998).
- Shimon & Rephaeli (2003) showed that all approaches are equivalent and derived an "elegant" analytic formula including multiple scattering and kinematic effects.
- Going to higher order in $\tau$ (Itoh & Nozawa 2004)
As can be easily seen...

\[ \Delta n_t = \tau(\Theta F_1 + \Theta^2 F_2 + \Theta^3 F_3 + \Theta^4 F_4 + \Theta^5 F_5 + \Theta^6 F_6 + \Theta^7 F_7 + \Theta^8 F_8) \]
\[ + \tau^2(\Theta^2 F_9 + \Theta^3 F_{10} + \Theta^4 F_{11} + \Theta^5 F_{12}). \]

\[ \frac{\Delta n}{\tau} = -\beta_c \mu_c \left( \frac{1}{2} A_1 + \Theta F_{13} + \Theta^2 F_{14} + \Theta^3 F_{15} + \Theta^4 F_{16} \right) \]
\[ + \beta_c^2 F_2(\mu_c)(F_{17} + \Theta F_{18} + \Theta^2 F_{19} + \Theta^3 F_{20} + \Theta^4 F_{21}) \]
\[ + \beta_c^2 \left( \frac{1}{3} F_1 + \Theta F_{22} + \Theta^2 F_{23} + \Theta^3 F_{24} + \Theta^4 F_{25} \right), \]

\[ F_1 = 2A_1 + \frac{1}{4} A_2, \]
\[ F_2 = 5A_1 + \frac{47}{8} A_2 + \frac{21}{20} A_3 + \frac{7}{160} A_4, \]
\[ F_3 = \frac{15}{4} A_1 + \frac{1023}{32} A_2 + \frac{217}{10} A_3 + \frac{329}{80} A_4 + \frac{11}{40} A_5 + \frac{11}{1920} A_6, \]
\[ F_4 = -\frac{15}{4} A_1 + \frac{2505}{32} A_2 + \frac{3549}{80} A_3 + \frac{14253}{160} A_4 + \frac{9297}{560} A_5 + \frac{12059}{8960} A_6 + \frac{1}{21} A_7 + \frac{1}{1680} A_8, \]
\[ F_5 = \frac{135}{64} A_1 + \frac{30375}{512} A_2 + \frac{62391}{80} A_3 + \frac{614727}{640} A_4 + \frac{124389}{320} A_5 + \frac{355703}{5120} A_6 + \frac{2071}{336} A_7 + \frac{1875}{6720} A_8 + \frac{11}{1792} A_9 + \frac{11}{215040} A_{10}. \]

\[ x_0 = 3.830016(1 + 1.120594\Theta + 2.078258\Theta^2 - 80.748072\Theta^3 + 1548.250996\Theta^4 + 0.800424\tau\Theta \]
\[ + 1.183073\tau\Theta^2). \]
Importance of rel. Corrections (Itoh et al. 2003)

\[ k_B T_e = 15 \text{keV} \]

\[ \frac{\Delta I}{y} \]

\[ X \]

- numerical results
- \( O(\theta^5) \)
- \( O(\theta^4) \)
- \( O(\theta^3) \)
- \( O(\theta^2) \)
- \( O(\theta) \)
Importance of rel. Corrections (Itoh et al. 2003)

$k_B T_e = 20\text{keV}$

\[ \frac{\Delta I}{y} \]

\[ \theta^5 \]
\[ \theta^6 \]
\[ \theta^7 \]
\[ \theta^8 \]
\[ \theta^9 \]
Importance of rel. Corrections (Itoh et al. 2003)
Polarisation

- Incident unpolarized radiation will become linearly polarized if it has a finite quadrupole moment.
- Polarisation effects are much weaker than effects on the total intensity and will not be measurable in the near future.
Systematics in Observations

- The SZE signal is relatively weak: $\sim 100\mu K$ at a given frequency
- The measurement is completely dominated by systematics
  $\Rightarrow$ All SZE measurements are differential
- The most important sources of systematical errors are given by:
  1. Emission of the Earth’s atmosphere $\sim 3K$
  2. Ground noise
  3. Radio sources (including grav. lensing)
  4. Detector calibration
  5. CMB-anisotropies (makes measuring kSZE difficult, one needs to use rel. corrections)
Single Dish Observations

- Pioneering work by Birkinshaw with the OVRO 40m telescope (1978, 1991)
- To control atmospheric and ground noise:
  1. Position-switching schemes (dish and secondary mirror)
  2. Beam-switching schemes
- Use of bolometric detectors
  1. High sensitivity
  2. Array operation for element differencing
  3. Multiple-band observation
- External radio-source monitoring
While single-dish telescopes allow large sky surveys, interferometers allow high resolution SZE maps due to largely improved systematics.

Radio arrays can be adjusted to observe particular scales by changing their baseline. Due to interferometry they are only sensitive to spatial frequencies near $B/\lambda$.

Because of this SZEs can be masked by going to large baselines and radio sources can be identified separately.

Arrays achieve extremely low systematics:

1. Correlations between pairs of telescopes are calculated $n(n-1)/2$ measurements.
2. Interferometers do not respond to constant background levels (const. atmosphere, ground noise, CMB background).
3. Gradients in the atmosphere become uncorrelated with baselines longer than a few meters.
The Useful Ones

OVRO

BIMA
The Special One
Results (Carlstrom 1999)

[Images of various galaxy clusters with contour maps and color-coded distributions indicating density or temperature variations.]

ICM: Sunyaev-Zel'dovich Effects
Results (Benson 2003)
<table>
<thead>
<tr>
<th>Cluster</th>
<th>Date</th>
<th>$\Delta$R.A. (arcsec)</th>
<th>$y_0 \times 10^4$</th>
<th>$v_{pec}$ (km s$^{-1}$)</th>
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</thead>
<tbody>
<tr>
<td>A2261</td>
<td>1999 Mar</td>
<td>$6.4^{+18.6}_{-19.5}$</td>
<td>$7.41^{+1.95}_{-1.98}$</td>
<td>$-1575^{+1500}_{-975}$</td>
</tr>
<tr>
<td>A2390</td>
<td>2000 Nov</td>
<td>$-4.8^{+18.1}_{-19.1}$</td>
<td>$1.72^{+1.01}_{-0.76}$</td>
<td>$+1900^{+6225}_{-2650}$</td>
</tr>
<tr>
<td>Zw 3146</td>
<td>2000 Nov</td>
<td>$11.4^{+30.9}_{-30.9}$</td>
<td>$3.62^{+1.83}_{-2.52}$</td>
<td>$-400^{+3700}_{-1925}$</td>
</tr>
<tr>
<td>A1835</td>
<td>1996 Apr</td>
<td>$28.0^{+16.0}_{-15.0}$</td>
<td>$7.66^{+1.64}_{-1.66}$</td>
<td>$-175^{+1675}_{-1275}$</td>
</tr>
<tr>
<td>Cl 0016</td>
<td>1996 Nov</td>
<td>$6.2^{+34.5}_{-37.4}$</td>
<td>$3.27^{+1.45}_{-2.86}$</td>
<td>$-4100^{+2650}_{-1625}$</td>
</tr>
<tr>
<td>MS 0451</td>
<td>1996 Nov</td>
<td>$-15.5^{+26.0}_{-24.0}$</td>
<td>$3.20^{+1.61}_{-1.61}$</td>
<td>$+175^{+5750}_{-2625}$</td>
</tr>
<tr>
<td></td>
<td>1997 Nov</td>
<td>$12.0^{+10.0}_{-11.0}$</td>
<td>$2.07^{+0.70}_{-0.72}$</td>
<td>$+1775^{+3900}_{-2150}$</td>
</tr>
<tr>
<td></td>
<td>2000 Nov</td>
<td>$-21.5^{+21.0}_{-19.0}$</td>
<td>$3.17^{+0.86}_{-0.88}$</td>
<td>$-300^{+1950}_{-1275}$</td>
</tr>
<tr>
<td>Combined fit</td>
<td></td>
<td></td>
<td>$2.84^{+0.52}_{-0.52}$</td>
<td>$+800^{+1525}_{-1125}$</td>
</tr>
</tbody>
</table>
SZE and Cosmology: The Hubble Constant

- $\Delta T_{SZE} \propto \int dl \ n_e T_e$
- $S_X \propto \int dl \ n_e^2 \Lambda_{eH}$
  with X-Ray cooling function $\Lambda_{eH}$

By eliminating electron density

$$\Rightarrow \quad D_A \propto \frac{(\Delta T_0)^2 \Lambda_{eH0}}{S_{X0} T_{e0}^2} \frac{1}{\Theta_c}$$

- $\Theta_c$ is the characteristic scale of the cluster-density model
- What we can measure is the characteristic scale of the cluster in the sky $\Theta_{c}^{\text{sky}}$
- By assuming $\langle \Theta_c / \Theta_{c}^{\text{sky}} \rangle = 1$ and $\sqrt{\langle n_e^2 \rangle} = \langle n_e \rangle$ we can measure the angular diameter distance of the cluster.

![Graph showing angular diameter distance $D_A$ versus redshift $z$](image)
The gas mass of a cluster can be measured directly through tSZ observations if the electron temperature is known.

The total gravitating mass is obtained through lensing or by the assumption of hydrostatic equilibrium.

Determining the gas fraction for a large region of the cluster provides a fair estimate for the universal baryon mass fraction

\[ f_B \equiv \frac{\Omega_B}{\Omega_M} \]

\( \Omega_B \) can be obtained by Big-Bang nucleosynthesis, D/H measurements in Lyman \( \alpha \) clouds or CMB primary anisotropies analysis.

Therefore an estimate for \( \Omega_M \) with the help of the SZE is also possible.
Galaxy Clusters produce a significant SZ signal.

Cluster detection by SZE is not limited at high redshift as e.g. X-Ray surveys.

Thousands of clusters in a wide redshift range should be detected by large SZE surveys (Planck).

SZE is a powerful tool to observe the high redshift universe.

source-count against redshift plots are a powerful measure of cosmological parameters.